Precalculus Learning Goals - Week 14

This week we're going to review. We've covered a lot in this course. Want to see how much? Here's all the material we've covered. But first, let's revisit the goals of the entire course. By the end of the course, students should be able to:

- **Understand that solving equations is just applying valid rules through enlightened or strategic experimentation to rewrite expressions in a simpler form.**
- **Use functions to realize that mathematics involves translating between numbers, words, formulas and pictures.**
- **Produce and evaluate the validity of a logical argument.**

The sections we covered were:

Algebra Review

- *Creatively and correctly apply the rules of algebra, and justify steps taken.*
- *Recognize and disprove an incorrect application of the rules of algebra.*
- *Apply enlightened guessing to follow common algebraic procedures.*

Learning Objectives.

- Perform arithmetic operations on real numbers and variables.
- Recognize when parentheses are needed to make an expression unambiguous.
- Apply rules of algebra (arithmetic, order of operations, distributive law, rules of exponents, etc.) to rewrite expressions.
- Factor quadratics.
- Explain what the solution set to an equation or inequality is.
- Solve simple algebraic equations through algebraic operations and by dividing into cases (including equations involving absolute value).
- Solve simple algebraic inequalities through algebraic operations and by dividing into cases (including inequalities involving absolute value).
- Use the "Zero-Factor Theorem" to solve equations through factoring.
- Check solutions to an equation or inequality.
- Use interval notation to define sets of real numbers.
- Explain the difference between intersections and unions of sets.
- Identify mistakes in a series of algebraic steps.
- Compute the distance between points in the plane.
- Identify sets of points in the plane described by words, sets, or equations.
- Transition between verbal and symbolic descriptions of circles.

Functions - Properties and Examples

- *Be comfortable with the language, notation, and pictures of functions, as well as be able to translate between them.*
- *Know several examples of functions and their basic properties, both mathematical and "real-world."*

- *Be able to generate new functions from old through the standard function operations.*

Learning Objectives.

- Define *function, domain, range, independent variable, dependent variable.*
- Read and write function notation, including piecewise function notation.
- Discuss the difference between a relation and a function.
- Identify whether a relation is a function or not based on a formula, graph, table, written description, or other presentation.
- Evaluate a function at a given input based on a formula, graph, table, written description, or other presentation.
- Given formula, graph, table, written description, or other presentation of a function, write the function in another form.
- Compute the (implied) domain and range of a function based on a formula, graph, table, written description, or other presentation (only requiring simple algebraic techniques).
- Define, graph, and state the domain and range of linear functions, quadratic functions, absolute value functions (as piecewise functions), root, and exponential functions.
- Describe how simple graph transformations affect the shape of the graph of a function (translations, compressions, stretches, reflections).
- Compute an arithmetic combination of or composition of two or more functions based on a formula, graph, table, written description, or other presentation.
- Write a function as the composition of two simpler functions.
- Construct a rule to define the domain of a composition of functions based on the domains of the individual functions.
- Define *1-1* or *injective* functions and *inverse* functions.
- Explain why a function must be 1-1 in order to have an inverse.
- Determine if a given function has an inverse based on a formula, graph, table, written description, or other presentation.
- Compute the inverse of a function algebraically.
- Compare the graphs of a function and its inverse.

Linear Functions

- *Given the equation of a line, sketch its graph, and vice versa.*

Learning Objectives.

- Given the equation of a line, state points on the line and sketch its graph.
- Compute the slope of a line given two points or a graph.
- Compute the y-intercept of a line given two points or a graph.
- Describe how the slopes of perpendicular lines are related.

Quadratic Functions

- *Solve any quadratic equation with real solutions.*

- *Transition between formulas and graphs of quadratic functions, and describe their basic properties.*

Learning Objectives.

- Factor quadratics of the form $ax^2 + bx + c$.
- State and apply the quadratic formula.
- Complete the square.
- Apply graph transformations to graph any quadratic function, given its formula.
- Solve application problems by modeling and then maximizing or minimizing a quadratic function.

Logarithmic Functions

- *Define logarithms.*
- *Compute with logarithms, including using rules of logarithms.*
- *Know major properties of graphs of logarithms.*

Learning Objectives.

- Define a logarithmic function as the inverse of an exponential function.
- Compute values of logarithmic functions (or estimate for "not nice" numbers).
- State the rules of logarithms.
- Use rules of logarithms to rewrite expressions.
- Convert a logarithmic equation into an exponential equation and vice versa.
- Use the change of base formula to rewrite expressions and compute logarithms.

Polynomial Functions

- *Know polynomial terminology and basic properties.*
- *Divide polynomials.*
- *Understand and apply the Intermediate Value Theorem.*

Learning Objectives.

- Define *polynomial, root, degree, coefficient, leading term, end behavior.*
- Find the quotient of two polynomials.
- Define *continuous* function intuitively.
- State the Intermediate Value Theorem.
- Apply the Intermediate Value Theorem to find zeros.
- Derive implications of the Intermediate Value Theorem.

Rational and Algebraic Functions

- *Understand how asymptotes and holes can arise with rational and algebraic functions.*

Learning Objectives.

- Find the holes of a rational function through factoring.
- Find the end behavior of a rational function.
- Find the vertical asymptotes of a rational function.
- Use "as $x \to a^{\pm}$, $f(x) \to b$" notation appropriately, for $a, b \in (-\infty, \infty)$.
- Describe vertical asymptotes of an algebraic function.
- Describe the end behavior of an algebraic function.

Trigonometry

- *Transition between interpretations of trig functions on triangles, the unit circle, and as graphs.*
- *Compute all trig and inverse trig functions for common values.*
- *Define inverse trig functions and explain their domain and range.*
- *Use trig functions to solve for missing quantities involving triangles and model periodic motion.*
- *Use trigonometric identities to simplify and rewrite expressions.*

Learning Objectives.

- Describe angles on the unit circle – locations and coordinates of common angles (multiples of $\pi/6$ and $\pi/4$), and approximate locations and coordinates of nonstandard angles.
- Explain the relationship between degrees and radians, and develop a conversion formula.
- Solve for missing pieces of information on the unit circle (e.g., given sine and quadrant, find cotangent).
- Define the six basic trigonometric functions on triangles and for any angle in the unit circle.
- Explain why trig functions are periodic.
- State the domain and range of trig functions.
- Graph trig functions, including with shifts and scaling.
- Define *period*, *amplitude*, and *periodic function*.
- Use trig functions to model periodic behavior.
- Relate a periodic graph to a real-world situation.
- Solve basic trig equations, including with factoring.
- Define inverse trig functions.
- Compute inverse trig functions.
- Use inverse trig functions as necessary to express solutions to trig equations.
- Describe the domain of inverse trig functions.
- Graph inverse trig functions.
- Evaluate or simplify a composition of trig and inverse trig functions.
- Solve more complicated trig equations.
- Explain what trigonometric identities are.
- Derive new trigonometric identities from old ones.
- Use trigonometric identities to solve trig equations.
- Use trigonometric identities to compute trig functions evaluated at nonstandard angles.

- *Graph exponential functions and identify their major features.*
- *Understand the significance of exponential functions and their major differences from other types of functions.*
- *Solve equations with exponentials and logarithms.*
- *Use exponential and logarithmic functions to model real-world phenomena.*

Learning Objectives.

- Define *exponential function*, *natural base*.
- State the domain and range of an exponential function.
- Graph an exponential function (including with graph transformations).
- Identify an exponential function (formula) based on its graph.
- Compare graphs and rate of growth of exponential functions with different bases.
- Describe the difference between exponential growth and linear or polynomial growth.
- Identify the horizontal asymptote of an exponential function, and explain why it exists.
- Describe why we only consider bases between 0 and 1, and greater than 1.
- Solve equations involving logarithms and exponentials.
- Solve equations involving logarithms and exponentials.
- Given two points, find an exponential curve of the form $P(t) = Ce^{kt}$ passing through those two points.
- Given a population growth, radioactive decay, compound interest, continuously compounded interest, or law of heating and cooling problem, extract relevant information to develop an exponential model and use that model to answer questions.